university of groningen

Exam Kaleidoscope

Ordinary Differential Equations

October 28th 2020   12.00-13.00

This exam is 'closed book'. It is NOT allowed to use a book or lecture notes. Only the use of a simple pocket calculator is allowed. Programmable calculators are not allowed, nor the use of electronic devices (tablet, laptop, etc.).

Always give a clear explanation of your answer. An answer without any computation will not be rewarded, so also copy the computations from your scratch paper.

Write your name and student number on each page!

Free points: [10]

1. In psychology, the learning curve is a function $P(t)$, where P denotes the performance of someone learning a skill, as a function of training time t. The derivative $P'(t)$ then obviously represents the rate at which the performance improves. When M is the maximum performance level, a model for learning is (with $k > 0$ a known constant)

$$P'(t) = k(M - P(t)).$$

When the maximum performance level increases in time, e.g. in case of a young child growing older, the model for learning becomes

$$P'(t) = k(M(t) - P(t)).$$

We will consider the case $k = 0.1$ and $M(t) = 120 + 10 * t * e^{-0.1\,t}$.
The boundary condition is $P(0) = 40$. In our model, t counts in years.

   (a) [4] What will be the value of $P(t)$ (approximately) after very long time ($t \to \infty$)?

   (b) [4] Determine the tangent line for the solution at $t = 0$, i.e. $\hat{P}(t) = P(0) + P'(0)\,t$.

   (c) [10] (1) Determine the time t_m when $M(t)$ reaches its maximum.
   (2) Does $P(t)$ reach its maximum value before t_m or after t_m?
   Evaluate both the diff. eqn. and the tangent line at t_m to explain your answer.

   (d) [17] Determine the solution $P(t)$ of the diff. eqn. with boundary condition $P(0) = 40$.

2. (a) [18] Determine the general solution $y(x)$ of the differential equation

$$y''(x) + 0.1\,y'(x) - 0.06\,y(x) = 21\,e^{-x} + 3.$$

   Hint: it is allowed to directly convert complex solutions (if any).

   (b) [7] Determine the solution given the boundary conditions $y(0) = 0$ and $y'(0) = 5$.

3. Consider the differential equation $\quad y'(x) = 2\,e^{x}\,y(x), \quad$ with bound. cond. $y(0) = 1$.

   (a) [15] (1)Use Euler's method, with steps of $\Delta x = 0.5$, to approximate $y(1.5)$.
   (2) Explain what happens to the numerical solution if the bound. cond. changes into $y(0) = 0$. Is this good or bad?

   (b) [15] Determine the 3rd order power-series solution, so terms up to x^3.
   Hint: setting up the Taylor series (directly) is the fastest way, but multiplication and comparison of series also works.

Total: [100]

Write your name and student number on each page!